Announcements

1) HW # 5 due Thursday, all on paper.

Example 1: Solve

$$y'' + (t-1)y' + y = 0$$

Suppose $y(t) = \sum_{n=0}^{\infty} a_n t^n$

with nonzero radius of convergence.

$$y(t) = \sum_{n=0}^{\infty} a_n t^n$$

$$y'(t) = \sum_{n=0}^{\infty} a_n n t^{n-1}$$

$$= \sum_{n=1}^{\infty} n a_n t^{n-1}$$

$$y''(t) = \sum_{n=1}^{\infty} n a_n (n-1) t^{n-2}$$

$$= \sum_{n=2}^{\infty} (n^2 - n) a_n t^{n-2}$$

Plugging into the equation

$$y'' + (t-1)y' + y = 0,$$

$$\sum_{n=2}^{\infty} (n^2-n) a_n t^{n-2} +$$

$$(t-1)\sum_{n=1}^{\infty} n a_n t^{n-1} + \sum_{n=0}^{\infty} a_n t^n$$

$$= 0.$$

$$(t-1)\sum_{n=1}^{\infty} n a_n t^{n-1}$$

$$= \sum_{n=1}^{\infty} n a_n t^{n} - \sum_{n=1}^{\infty} n a_n t^{n-1}$$

$(k=n-1)$

$$= \sum_{n=1}^{\infty} n a_n t^{n} - \sum_{k=0}^{\infty} (k+1) a_{k+1} t^{k}$$

$(\text{let } k=n)$

$$= -a_1 + \sum_{n=1}^{\infty} (n a_n - (n+1) a_{n+1}) t^{n}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1)a_n t^{n-2}$$

(let $k = n-2$)

$$= \sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2} t^{k}$$

(let $k = n$)

$$= \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} t^{n}$$